

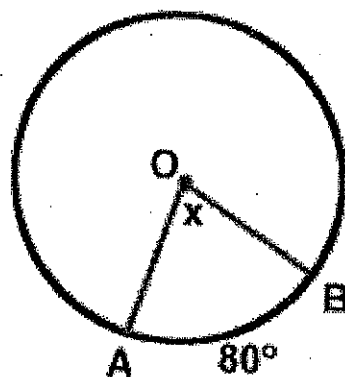
Formulas for Working with Angles in Circles:

(*Intercepted arcs* are arcs "cut off" or "lying between" the sides of the specified angles.)

1. Central Angle:

A central angle is an angle formed by two intersecting radii such that its vertex is at the center of the circle.

Central Angle =



$\angle AOB$ is a central angle.

Its *intercepted arc* is the minor arc from A to B.

$$m\angle AOB = 80^\circ$$

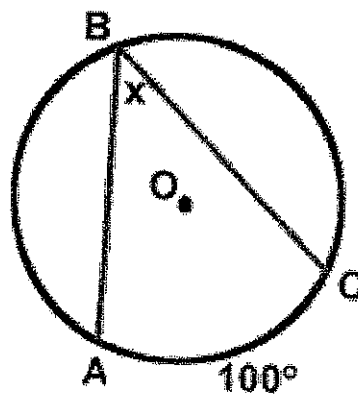
Theorem involving central angles:

In a circle, or congruent circles, congruent central angles have congruent arcs.

2. Inscribed Angle:

An inscribed angle is an angle with its vertex "on" the circle, formed by two intersecting chords.

Inscribed angle =



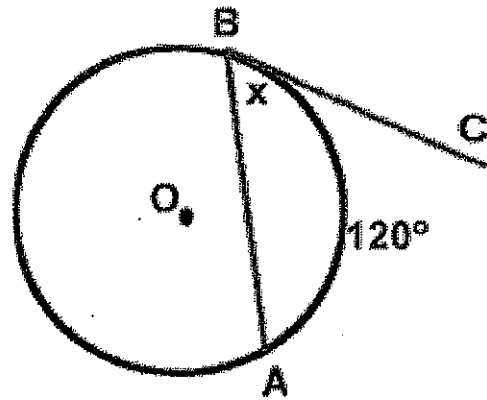
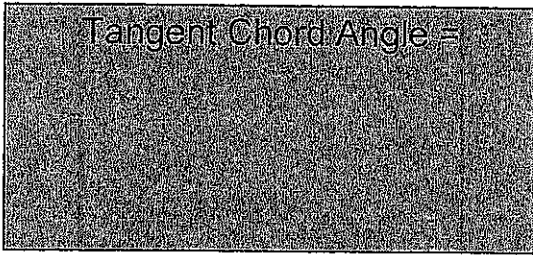
$\angle ABC$ is an inscribed angle.

Its *intercepted arc* is the minor arc from A to C.

$$m\angle ABC = 50^\circ$$

3. Tangent Chord Angle:

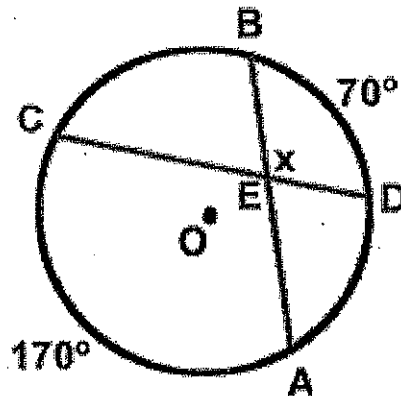
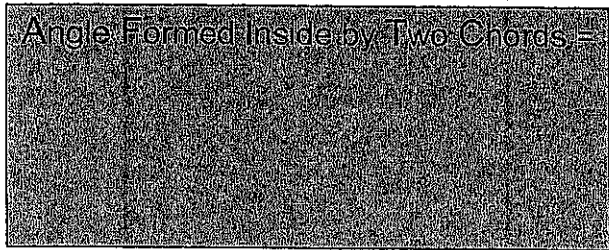
An angle formed by an intersecting tangent and chord has its vertex "on" the circle.



$\angle ABC$ is an angle formed by a tangent and chord.
Its *intercepted arc* is the minor arc from A to B .
 $m\angle ABC = 60^\circ$

4. Angle Formed Inside of a Circle by Two Intersecting Chords:

Remember: vertical angles are equal.



$\angle BED$ is formed by two intersecting chords.

Its *intercepted arcs* are \widehat{BD} and \widehat{CA} .

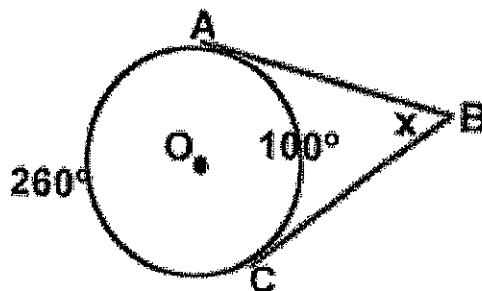
5. Angle Formed Outside of a Circle by the Intersection
of: "Two Tangents" or "Two Secants" or "a Tangent and a Secant".

The formulas for all THREE of these situations are the same
 Angle Formed Outside =

Two Tangents:

$\angle ABC$ is formed by two tangents intersecting outside of circle O .

The *intercepted arcs* are minor arc \widehat{AC} and major arc \widehat{AC} .

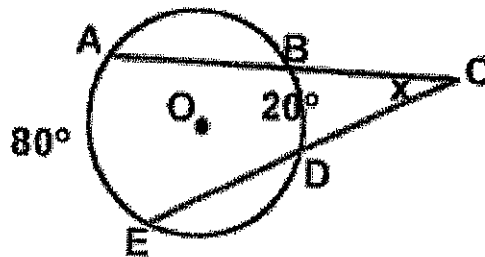


$$m\angle ABC = \frac{1}{2} \left(\overset{\text{major}}{m\widehat{AC}} - \overset{\text{minor}}{m\widehat{AC}} \right)$$

Two Secants:

$\angle ACE$ is formed by two secants intersecting outside of circle O .

The *intercepted arcs* are minor arcs \widehat{BD} and \widehat{AE} .

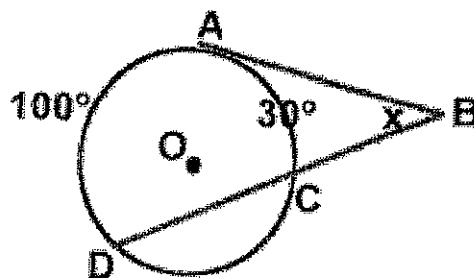


$$m\angle ACE = \frac{1}{2} \left(m\widehat{AE} - m\widehat{BD} \right)$$

a Tangent and a Secant:

$\angle ABD$ is formed by a tangent and a secant intersecting outside of circle O .

The *intercepted arcs* are minor arcs \widehat{AC} and \widehat{AD} .

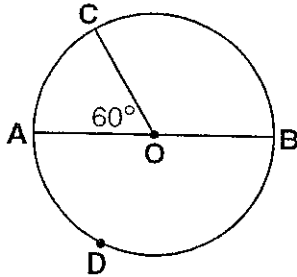


Name: _____ Date: _____

Central and Inscribed Angles

Questions 1 through 7 refer to the following:

In circle O below, \overline{AB} is a diameter and $m\angle AOC = 60^\circ$.

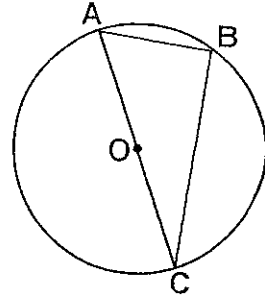


Find the value of the following:

- 1) $m\widehat{AC}$
- 2) $m\widehat{ABC}$
- 3) $m\angle COB$
- 4) $m\widehat{CB}$
- 5) $m\widehat{ACB}$
- 6) $m\widehat{ADB}$

7) $m\widehat{CAB}$

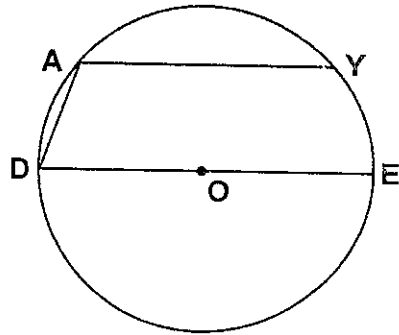
8) In the accompanying diagram of circle O, the ratio of \widehat{BC} to \widehat{AB} is 2:1.



What is $m\angle ACB$?

- | | |
|---------------|---------------|
| A) 45° | C) 90° |
| B) 60° | D) 30° |

9) In the accompanying diagram of circle O, chord \overline{AY} is parallel to diameter \overline{DOE} , \overline{AD} is drawn, and $m\widehat{AD} = 40^\circ$.



What is $m\angle DAY$?

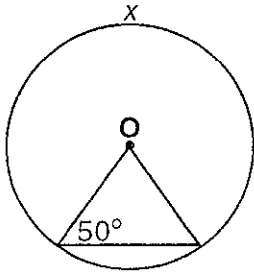
- | | |
|----------------|----------------|
| A) 110° | C) 130° |
| B) 90° | D) 150° |

Central and Inscribed Angles

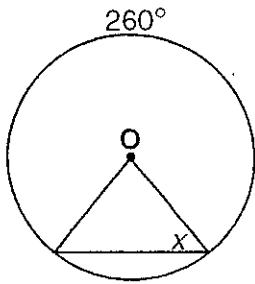
Questions 10 through 18 refer to the following:

For the given circle, find the value of x .

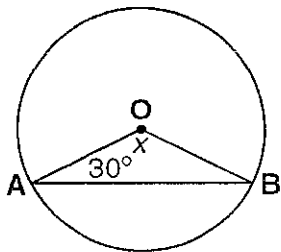
10)



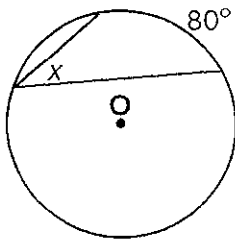
11)



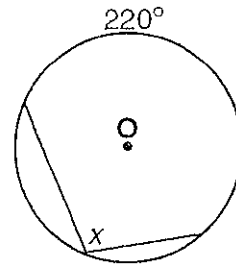
12)



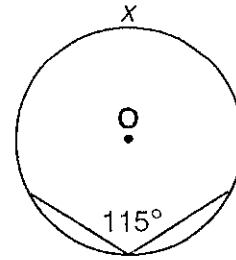
13)



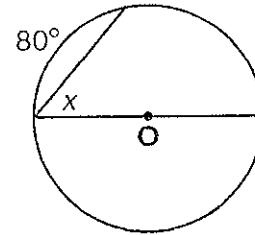
14)



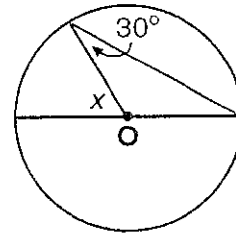
15)



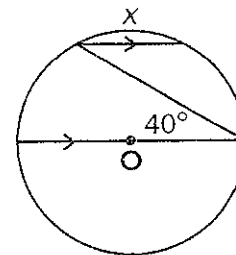
16)



17)



18)

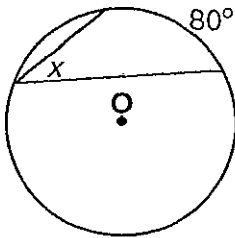


Practice with central and inscribed angles and angles formed by a chord and a tangent.

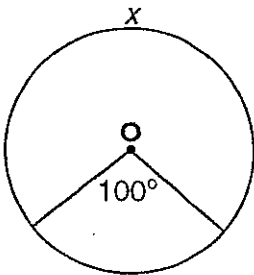
Questions 1 through 5 refer to the following:

For the given circle, find the value of x .

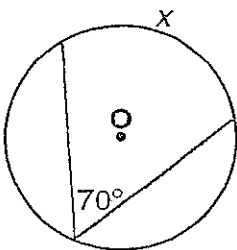
1)



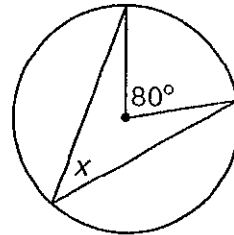
2)



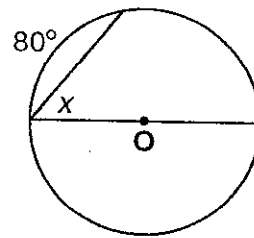
3)



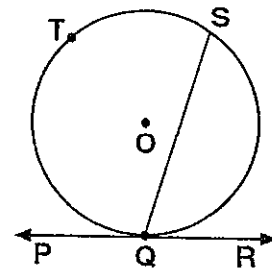
4)



5)

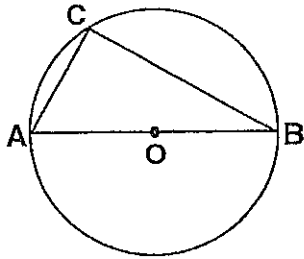


6)



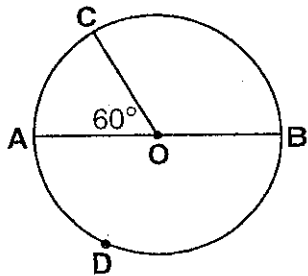
If $m\angle PQS = 122^\circ$, find $m\widehat{QS}$.

- 7) In the accompanying diagram, $\triangle ABC$ is inscribed in circle O and \overline{AB} is a diameter.



What is the number of degrees in $m\angle C$?

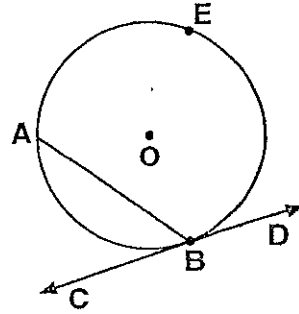
- A) 60° C) 30°
 B) 45° D) 90°
- d) In circle O below, \overline{AB} is a diameter and $m\angle AOC = 60^\circ$.



Find the value of the following:

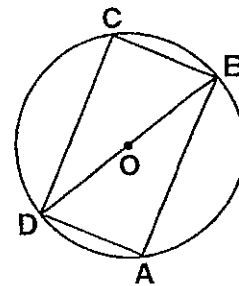
$$m\widehat{CB}$$

- 9) \overline{AB} is a chord in circle O and \overline{CD} is tangent at B .



If $m\widehat{AB} = 100^\circ$, find $m\angle ABC$, $m\widehat{AEB}$, and $m\angle ABD$.

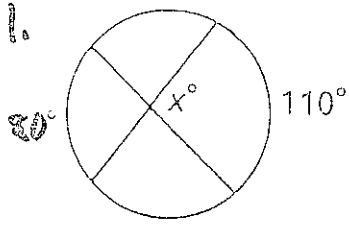
- 10) In the diagram below, rectangle $ABCD$ is inscribed in circle O .



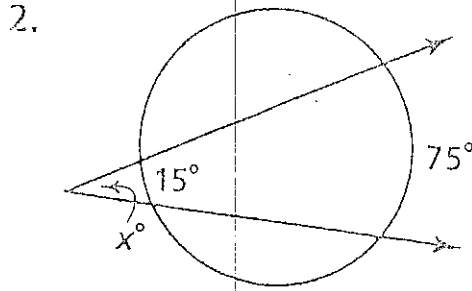
If $DB = 13$ and $AB = 12$, find DA .

classwork:

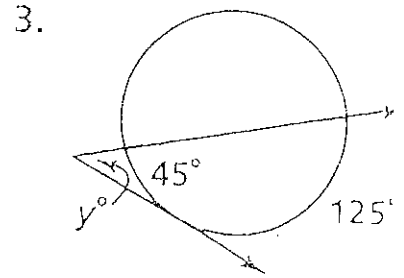
Angles formed by Tangents, Secants, and Chords



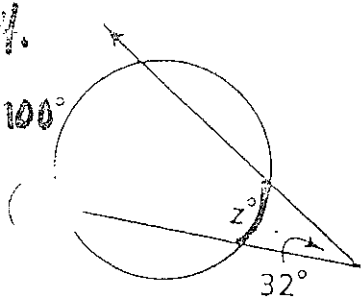
$X =$



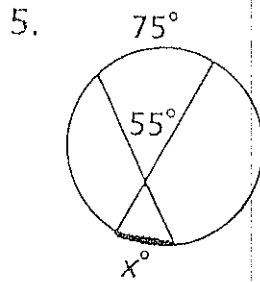
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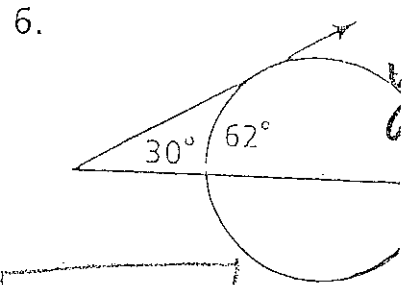
$y =$



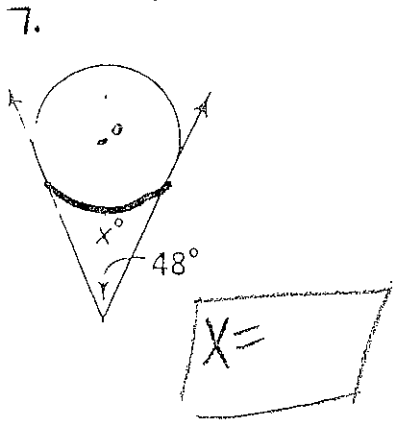
$z =$



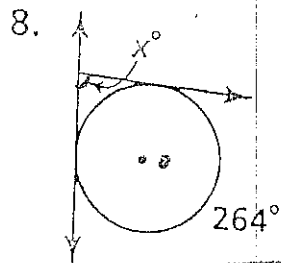
$X =$



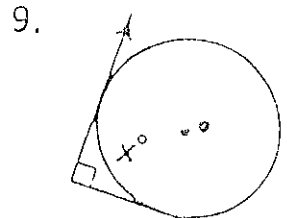
$y =$



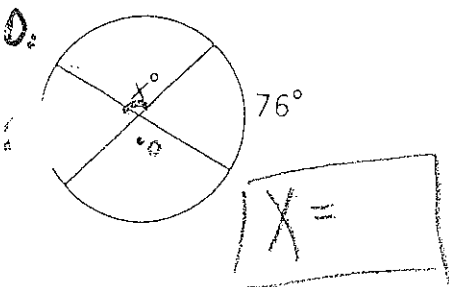
$X =$



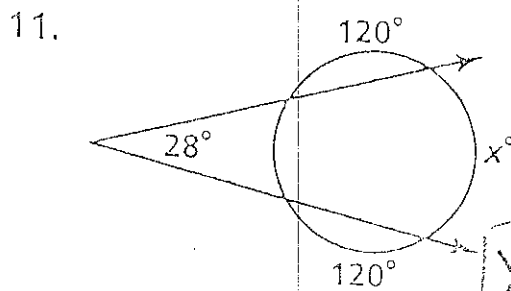
$X =$



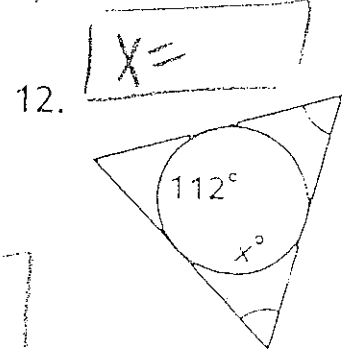
$X =$



$X =$



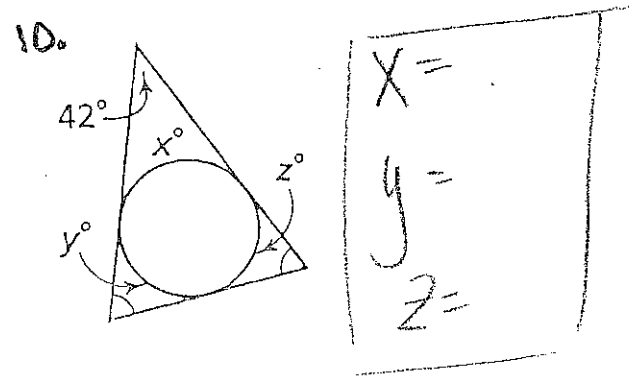
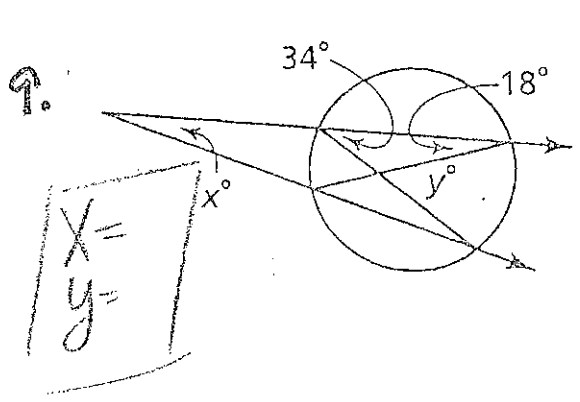
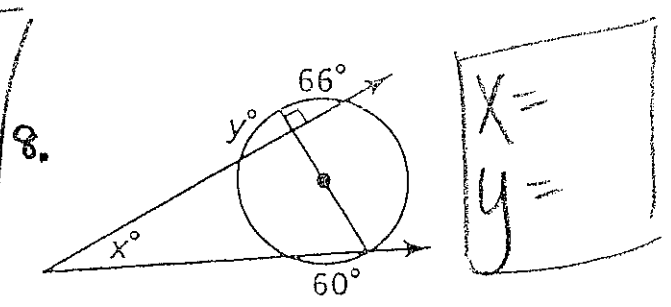
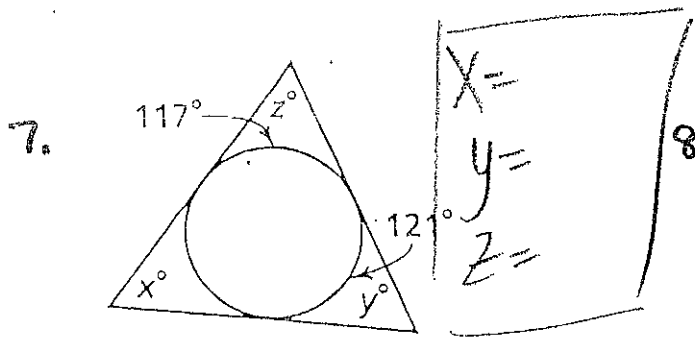
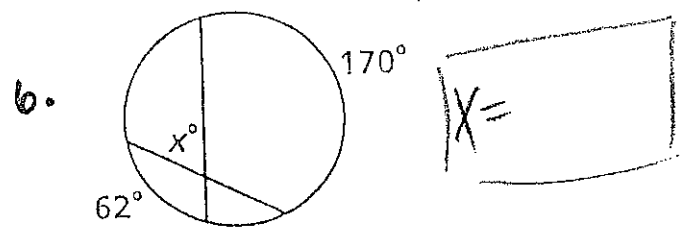
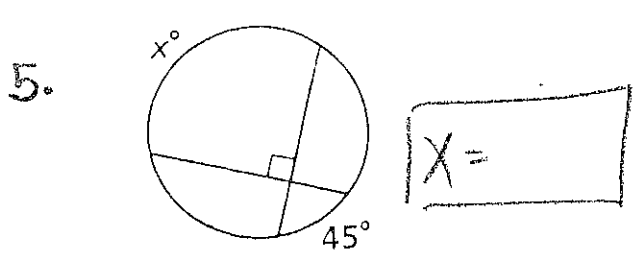
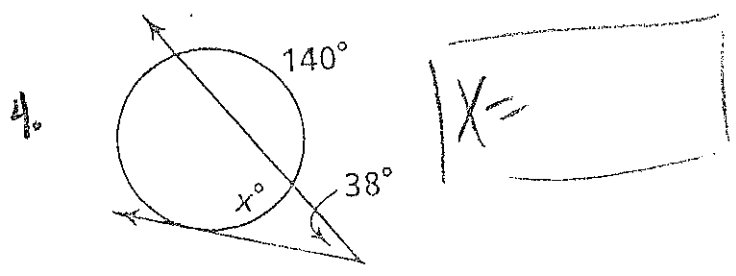
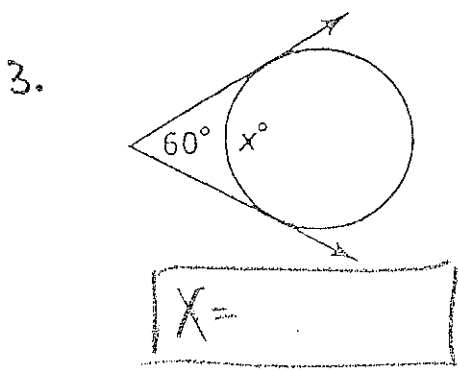
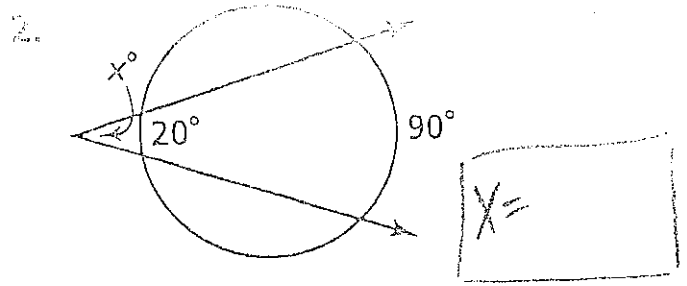
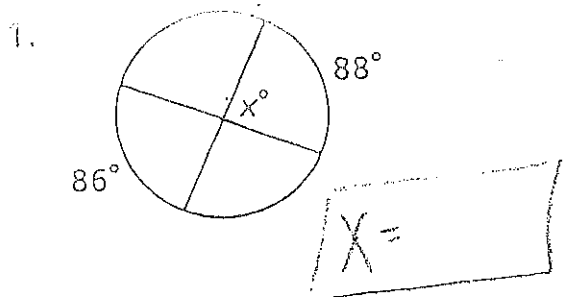
$X =$



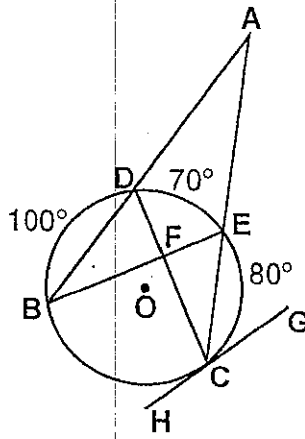
$X =$

Find the value of $x, y, \text{ or } z$

SHOW ALL WORK



- 11) In the accompanying diagram of circle O , \overline{ADB} and \overline{AEC} are secants, chords \overline{BE} and \overline{CD} intersect at F , tangent \overline{GH} intersects circle O at C , $m\widehat{BD} = 100^\circ$, $m\widehat{DE} = 70^\circ$, and $m\widehat{EC} = 80^\circ$.



Find:

a) \widehat{BC}

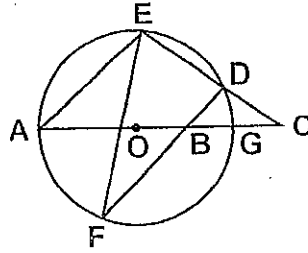
b) $m\angle BEC =$

c) $m\angle BDC =$

d) $m\angle GCE =$

e) $m\angle DCH =$

- 12) In the accompanying diagram of circle O , \overline{AE} and \overline{FD} are chords, \overline{AOBG} is a diameter and is extended to C , \overline{CDE} is a secant, $\overline{AE} \parallel \overline{FD}$, and $m\widehat{AE}:m\widehat{ED}:m\widehat{DG} = 5:3:1$.



Find:

a) value of x

b) $m\widehat{AFG}$

c) $m\angle EFD$

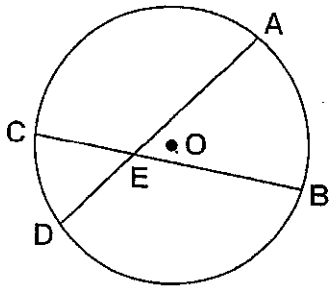
d) $m\angle EAG$

e) $m\angle AEF$

Name: _____

Date: _____
Circles and Angles

- 1) In the accompanying diagram of circle O, $m\widehat{AB} = 64^\circ$ and $m\angle AEB = 52^\circ$.

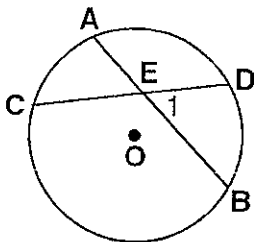


What is the measure of \widehat{CD} ?

- A) 104° C) 80°
B) 52° D) 40°

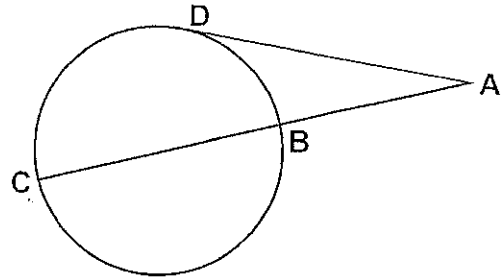
Questions 2 and 3 refer to the following:

In circle O below, chords \overline{AB} and \overline{CD} intersect at E.



- 2) If $m\widehat{AC} = 32^\circ$ and $m\widehat{DB} = 52^\circ$, what is $m\angle 1$?
A) 26° C) 84°
B) 42° D) 10°
- 3) If $m\angle 1 = 45^\circ$ and $m\widehat{AC} = 35^\circ$, what is $m\widehat{DB}$?
A) 55° C) 10°
B) 73° D) 125°

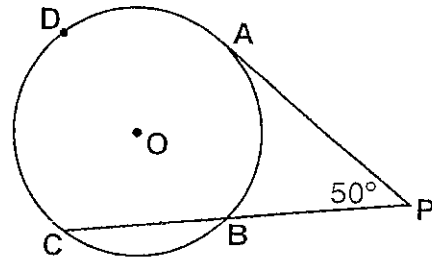
- 4) In the accompanying diagram, \overline{AD} is tangent to the circle at D and \overline{ABC} is a secant.



Find $m\angle A$ if $m\widehat{DC} = 120^\circ$ and $m\widehat{CB} = 170^\circ$.

- A) 70° C) 35°
B) 50° D) 25°

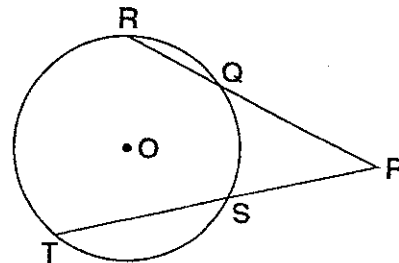
- 5) In the accompanying diagram, tangent \overline{PA} and secant \overline{PBC} are drawn to circle O.



If $m\widehat{ADC}$ is twice $m\widehat{AB}$ and $m\angle P$ is 50° , what is $m\widehat{AB}$?

- A) 200° C) 25°
B) 50° D) 100°

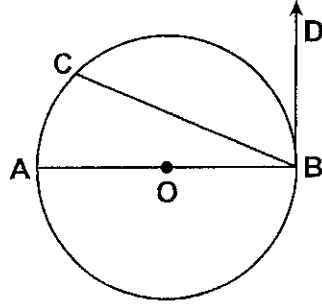
- 6) In the diagram below, secant segments \overline{PR} and \overline{PT} intersect at P.



If $\angle P = 42^\circ$ and $m\widehat{RT} = 148^\circ$, what is the measure of \widehat{QS} ?

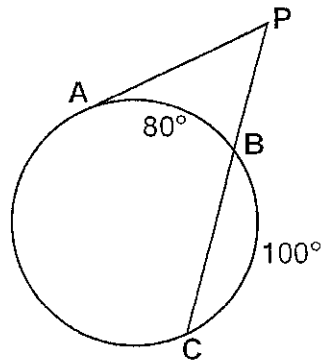
- A) 64° C) 74°
B) 106° D) 53°

- 7) In the accompanying diagram, \overline{BD} is tangent to circle O at B, \overline{BC} is a chord, and \overline{BOA} is a diameter.



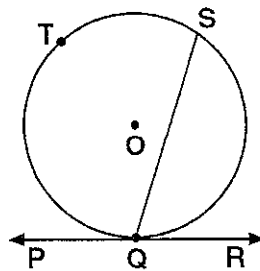
If $m\widehat{AC} : m\widehat{CB} = 1:4$, find $m\angle DEC$.

- 8) In the diagram below, \overline{PA} is tangent to the circle at A and \overline{PBC} is a secant.



If $m\widehat{AB} = 80^\circ$ and $m\widehat{BC} = 100^\circ$, what is $m\angle APB$?

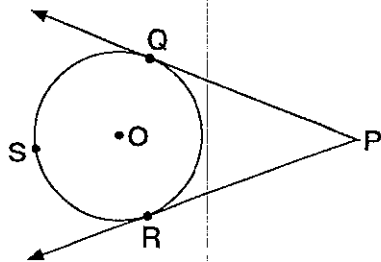
Questions 9 and 10 refer to the following:



- 9) If $m\widehat{QS} = 120^\circ$, find $m\angle SQR$.
- 10) If $m\angle SQR = 52^\circ$, find $m\widehat{QTS}$.

Questions 11 and 12 refer to the following:

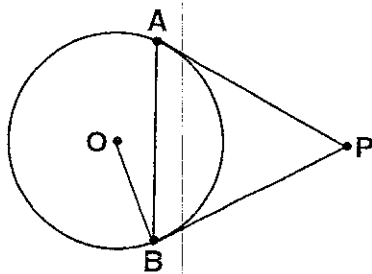
Tangents \overline{PQ} and \overline{PR} are drawn to circle O from P in the figure below.



11) If $m\widehat{QSR} = 230^\circ$, find $m\angle P$.

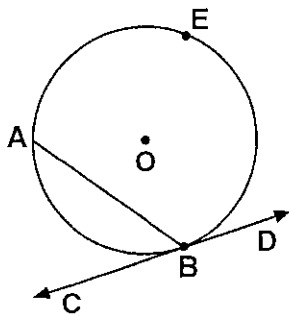
12) If $m\angle P = 30^\circ$, find $m\widehat{QR}$.

13) In the diagram below, \overline{PA} and \overline{PB} are tangent segments to circle O from P .



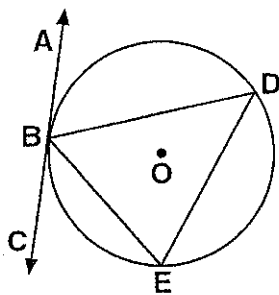
If $m\angle P = 42^\circ$, find $m\angle ABO$.

- 14) \overline{AB} is a chord in circle O and \overline{CD} is tangent at B.



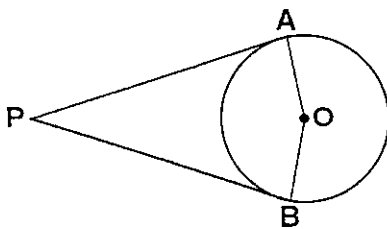
If $m\angle ABC = 56^\circ$, find $m\widehat{AB}$, $m\widehat{AEB}$, and $m\angle ABD$.

- 15) In the diagram below, \overline{AC} is tangent to circle O at B.



If $m\angle ABD = 70^\circ$ and $m\angle CBE = 42^\circ$, find the measure of each angle of $\triangle BDE$.

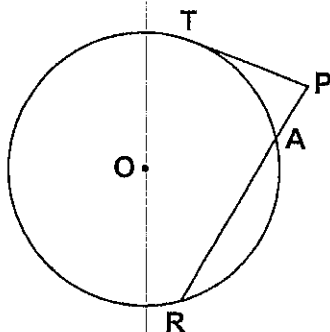
- 16) In the diagram below, \overline{PA} and \overline{PB} are tangent to circle O from P.



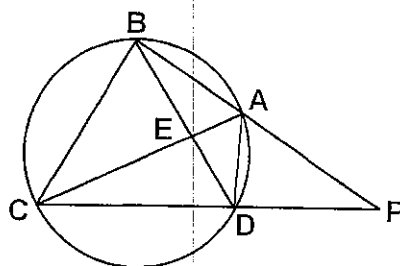
If $m\angle P = (x - 20)^\circ$ and $m\angle AOB = (2x + 50)^\circ$, find

- $m\angle AOB$
- $m\angle APB$
- $m\angle APO$

- 17) The accompanying diagram shows a circular machine part that has rods \overline{PT} and \overline{PAR} attached at points T, A, and R, which are located on the circle; $m\widehat{TA}:m\widehat{AR}:m\widehat{RT} = 1:3:5$; $RA = 12$ centimeters; and $PA = 5$ centimeters.



- (a) Find the measure of $\angle P$, in degrees. [Show all work.]
- (b) Find the length of rod \overline{PT} , to the nearest tenth of a centimeter. [Show all work.]
- 18) In the accompanying diagram, quadrilateral ABCD is inscribed in the circle, diagonals \overline{AC} and \overline{BD} intersect at E, sides \overline{BA} and \overline{CD} are extended to P, $m\widehat{AD}:m\widehat{DC} = 1:2$, $m\widehat{ABC} = 234^\circ$, and $m\widehat{BC}$ is 54 less than twice $m\widehat{AB}$.



Find:

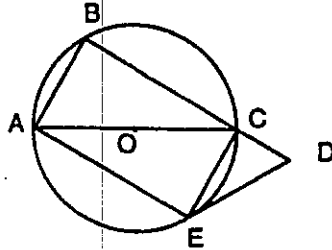
- (a) $m\widehat{AB}$
- (b) $m\angle ABD$
- (c) $m\angle BPC$
- (d) $m\angle AEB$
- (e) $m\angle ADP$

"Big Circle Problem"

Name: _____

Date: _____

In the accompanying diagram of circle O , $m\widehat{AB} : m\widehat{BC} = 1 : 2$;
 diameter \overline{CA} and chord \overline{AE} are drawn;
 chord \overline{EC} is parallel to chord \overline{AB} ;
 chord \overline{BC} is extended through C to D ;
 and tangent \overline{DE} is drawn.



Find:

a) $\widehat{BC} =$

b) $\widehat{CE} =$

c) $\angle AEC =$

d) $\angle BCA =$

e) $\angle ACE =$

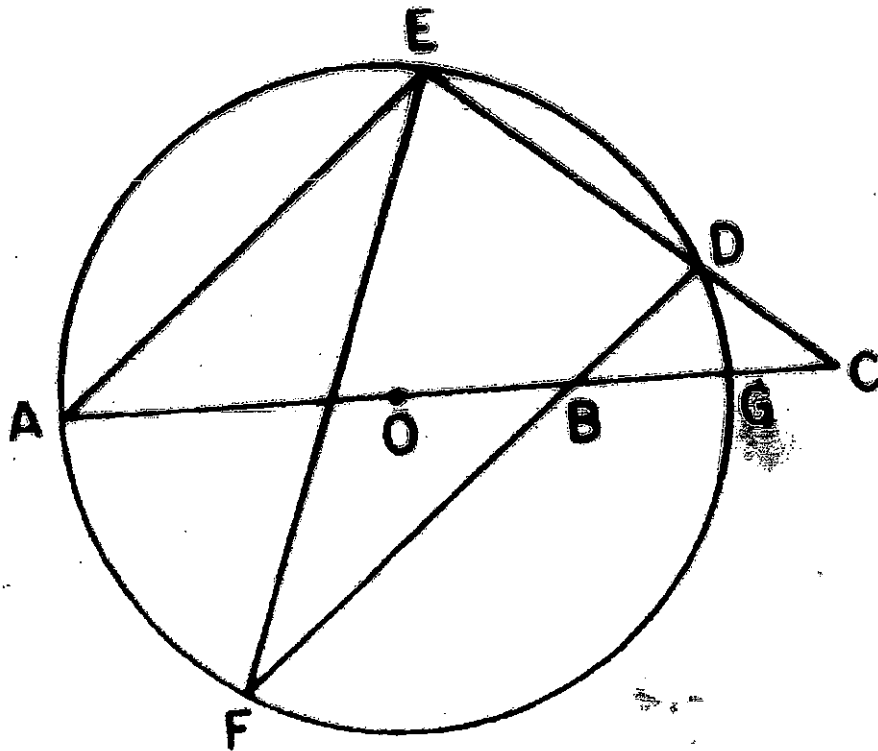
f) $\angle DCE =$

g) $\angle CED =$

In the accompanying diagram of circle O , \overline{AE} and \overline{FD} are chords, \overline{AOBG} is a diameter and is extended to C , \overline{CDE} is a secant, $\overline{AE} \parallel \overline{FD}$, and $m\widehat{AE} : m\widehat{ED} : m\widehat{DG} = 5 : 3 : 1$.

Find:

- $m\widehat{DG}$ [2]
- $m\angle AEF$ [2]
- $m\angle DBG$ [2]
- $m\angle DCA$ [2]
- $m\angle CDF$ [2]



Date _____

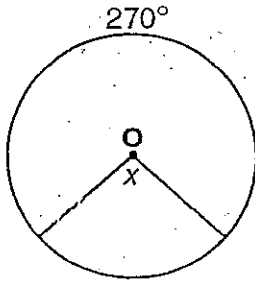
Circle Review

Name: _____

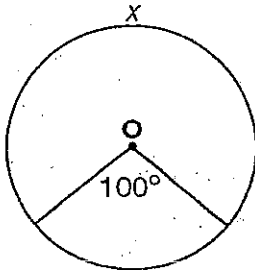
Questions 1 through 5 refer to the following:

For the given circle, find the value of x .

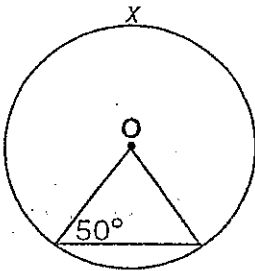
1)



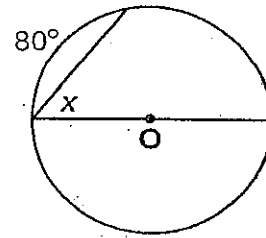
2)



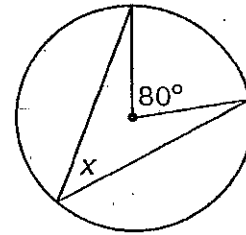
3)



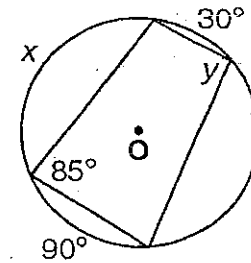
4)



5)

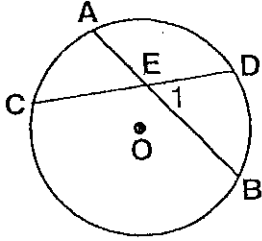


6) For the given circle, find the value of x and y .



Questions 7 through 9 refer to the following:

In circle O below, chords \overline{AB} and \overline{CD} intersect at E.

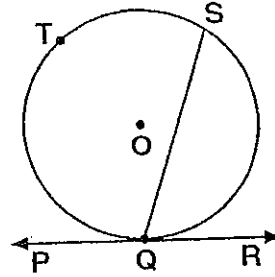


- 7) If $m\widehat{AC} = 30^\circ$ and $m\widehat{DB} = 50^\circ$, what is $m\angle 1$?
- A) 40° C) 25°
 B) 80° D) 10°

- 8) If $m\angle 1 = 50^\circ$ and $m\widehat{AC} = 40^\circ$, what is $m\widehat{DB}$?
- A) 10° C) 50°
 B) 60° D) 80°

- 9) If $m\angle 1 = 44^\circ$ and $m\widehat{DB} = 50^\circ$, what is $m\widehat{AC}$?
- A) 19° C) 70°
 B) 38° D) 10°

Questions 10 and 11 refer to the following:

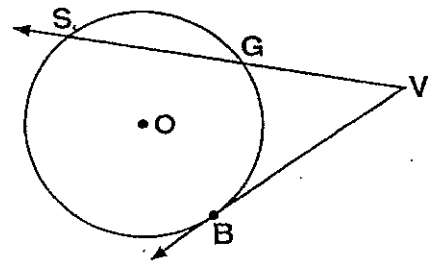


- 10) If $m\widehat{QS} = 120^\circ$, find $m\angle SQR$.

- 11) If $m\angle SQR = 52^\circ$, find $m\widehat{QT}$.

Questions 12 and 13 refer to the following:

In the diagram below, secant \overline{VS} and tangent \overline{VB} are drawn to circle O.

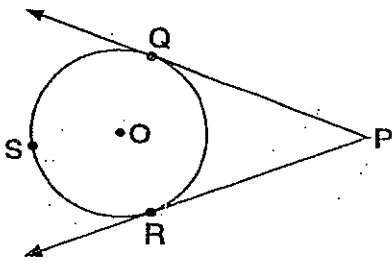


- 12) If $m\widehat{SB} = 170^\circ$ and $m\widehat{GB} = 90^\circ$, find $m\angle V$.

13) If $m\angle V = 38^\circ$ and $m\widehat{GB} = 92^\circ$, find \widehat{SB} .

Questions 14 and 15 refer to the following:

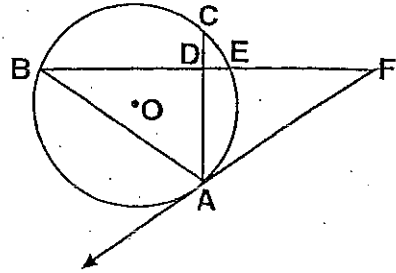
Tangents \overline{PQ} and \overline{PR} are drawn to circle O from P in the figure below.



14) If $m\widehat{QR} = 140^\circ$, find $m\angle P$.

15) If $m\angle P = 80^\circ$, find $m\widehat{QR}$.

16) In circle O , \overline{FA} is a tangent, \overline{FEDB} is a secant, \overline{ADC} and \overline{AB} are chords, $m\widehat{CE} = 40^\circ$, $m\widehat{AB} = 130^\circ$, and $m\angle CAB = 60^\circ$.



Find:

- (a) $m\widehat{BC}$
- (b) $m\angle EBA$
- (c) $m\angle ADE$
- (d) $m\angle F$
- (e) $m\angle FAC$

